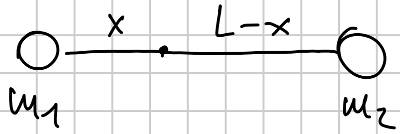
A.A. 2010/11 – 2° COMPITINO – 31/1/2011

Titolo nota 26/01/2011

1) 

a) $I = m_1 x^2 + m_2 (L-x)^2 =$

$= (m_1+m_2)x^2 + m_2 L^2 - 2m_2 L x$

b) $x_{cm} = \dfrac{0 \cdot m_1 + L \cdot m_2}{m_1+m_2} = \dfrac{m_2}{m_1+m_2} L$

$K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (\omega x_{cm})^2 = \frac{1}{2}\omega^2 L^2 \dfrac{m_1 m_2^2}{(m_1+m_2)^2}$

$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left[\omega(L-x_{cm})\right]^2 = \frac{1}{2}\omega^2 m_2 \cdot \left[\dfrac{m_1 L}{m_1+m_2}\right]^2 =$

$= \frac{1}{2}\omega^2 L^2 \dfrac{m_1^2 m_2}{(m_1+m_2)^2} \Rightarrow \dfrac{K_1}{K_2} = \dfrac{m_1 m_2^2}{m_1^2 m_2} = \dfrac{m_2}{m_1}$

Altra risoluzione:

$K_1 = \frac{1}{2} I_1 \omega^2 \qquad K_2 = \frac{1}{2} I_2 \omega^2 \qquad (\omega_1 = \omega_2 = \omega$, ovviamente$)$

$\dfrac{K_1}{K_2} = \dfrac{I_1}{I_2} = \dfrac{m_1 x_{cm}^2}{m_2(L-x_{cm})^2} = \dfrac{\cancel{m_1}}{\cancel{m_2}} \dfrac{m_2^{\cancel{2}}}{\cancel{(m_1+m_2)^2}} \cdot \dfrac{\cancel{(m_1+m_2)^2}}{m_1^{\cancel{2}}} = \dfrac{m_2}{m_1}$

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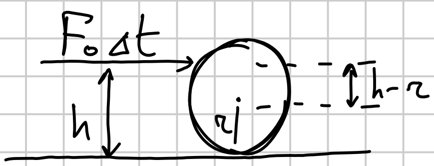
2) 

a) $\vec{J}_{angolare} = \vec{F}_0 \Delta t \cdot (h-r) = \Delta \vec{L}$

$\Rightarrow L_{finale} = I_{cm} \cdot \omega_0 = F_0 \Delta t (h-r)$

$\Rightarrow \omega_0 = \dfrac{F_0 \Delta t (h-r)}{I_{cm}} = \dfrac{F_0 \Delta t (h-r)}{\frac{2}{5} m r^2} = \dfrac{5 F_0 \Delta t (h-r)}{2 m r^2}$

$\vec{J}_{lineare} = \vec{F}_0 \Delta t = \Delta \vec{p} \Rightarrow p_{finale} = m V_0 = F_0 \Delta t$

$\Rightarrow V_0 = \dfrac{F_0 \Delta t}{m} \Rightarrow \omega_0 = 5 V_0 \dfrac{h-r}{2r^2}$

b) Per puro rotolamento (= rotolamento senza strisciare) sin dall'inizio deve valere $\omega_0 = \dfrac{V_0}{r}$

$\Rightarrow 5\dfrac{h-r}{2r^2} = \dfrac{1}{r} \Rightarrow h = \frac{7}{5} r$

3) $mg = F_p = F_G$ (superficie della Terra) $= \dfrac{G m M_T}{R_T^2}$

$$\Rightarrow \quad \frac{G M_T}{R_T^2} = g$$

Satellite geostazionario $\Rightarrow T = T_{\text{rotazione Terra}} = 1\,d = 86400\,s$

$F_{\text{centripeta sul satellite}} = F_G$ (satellite – Terra)

$$\cancel{m_{sat.}}\, \omega^2 \cdot d_{TS} = G \frac{\cancel{m_{sat.}} \cdot M_T}{d_{TS}^2}$$

$$\frac{4\pi^2}{T^2} d_{TS}^3 = G M_T = g R_T^2$$

$$\Rightarrow d_{TS} = \sqrt[3]{\frac{g R_T^2 T^2}{4\pi^2}} = \sqrt[3]{\frac{9.81 \frac{m}{s^2} \times 6.37^2 \times 10^{12}\, m^2 \times 8.64^2 \times 10^8\, s^2}{4 \times 3.14^2}} =$$

$$= \sqrt[3]{753.453 \times 10^{20}}\ m = 42.236 \times 10^6\ m = 42236\ km$$

$$E_{TOT} = \frac{1}{2} m v^2 - \frac{G m M_T}{d_{TS}} = -\frac{G m M_T}{2 d_{TS}} = -\frac{m g R_T^2}{2 d_{TS}} = -1.46 \times 10^{10}\ J$$

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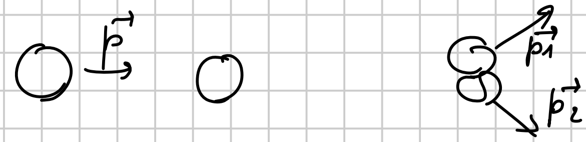
4)



Urto $\longrightarrow$ conservazione della quantità di moto $\qquad \vec{p} = \vec{p_1} + \vec{p_2}$

elastico $\longrightarrow$ conservazione della energia cinetica $\qquad K = K_1 + K_2$

$$\frac{p^2}{2m} = \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$$

$$\Rightarrow \begin{aligned} \vec{p} &= \vec{p_1} + \vec{p_2} \\ p^2 &= p_1^2 + p_2^2 \end{aligned}$$

$$p_1^2 + p_2^2 = p^2 = \vec{p} \cdot \vec{p} = (\vec{p_1} + \vec{p_2}) \cdot (\vec{p_1} + \vec{p_2}) = \vec{p_1} \cdot \vec{p_1} + \vec{p_2} \cdot \vec{p_2} + 2 \vec{p_1} \cdot \vec{p_2}$$

$$\Rightarrow p_1^2 + p_2^2 = p_1^2 + p_2^2 + 2 \vec{p_1} \cdot \vec{p_2} \quad \Rightarrow \quad \vec{p_1} \cdot \vec{p_2} = 0 \quad \Rightarrow \quad \vec{p_1} \perp \vec{p_2}$$